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Scaling of confined membranes and interfaces

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Abstract. We have carried out Monte Carlo studies of the probability distribution functions (PDFs) for models of two- and three-dimensional membranes and interfaces confined between two parallel repulsive walls separated by a distance D. For two-dimensional interfaces it is known that the position PDF $p(z)$, for conformally mapped binding potentials, scales and is characterized by a universal scaling function $p(z) \sim (\sin \pi z/D)^{\vartheta-1}$ (where ϑ is the short distance expansion critical exponent) for strong, weak and intermediate fluctation regimes. Our simulation studies show that for a variety of membrane models the PDF has the expected scaling $p(z) = U(z/D)/D$, and we find that the same parametrization of the membrane PDFs gives an excellent fit to the numerical data.

1. Introduction

The statistical mechanics of interfaces separating coexisting phases and the surfaces of complex fluid systems are interesting because of the large-scale fluctuations that may occur in the position of the surface at any point in space. In this paper we discuss the properties of model membranes and interfaces, focusing on the finite-size effects that occur when such a surface is confined between a pair of flat parallel walls. Related problems have been studied recently by Maggs *et al* (1989), Gompper and Kroll (1991), Fisher (1986) and Parry (1992a, b).

Quite generally the presence of the walls affects the statistical mechanics of the surface (interface or membrane) entropically by eliminating a large number of configurations which otherwise would cross the walls. This idea is central, for example, to the effective, fluctuation-induced steric repulsion force between nearby membranes first postulated by Helfrich (1978).

A central object of our study is the one-body probability distribution function (PDF) $p(z)$, defined such that $p(z)\,\mathrm{d}z$ is the probability of finding the surface between z and $z + \mathrm{d}z$. Following earlier studies (Parry *et al* 1991), we are interested in the case where the interface or membrane is confined exactly at a (fluctuation-dominated) wetting transition. This is equivalent to the situation in which the fluctuating surface simultaneously unbinds from both walls in the limit of infinite separation. Standard scaling arguments then suggest that $p(z)$ will take the scaling form $U(z/D)/D$, where D is the distance between the walls, and U is an appropriate scaling function. For interfaces in two dimensions this can easily be checked using analytical transfer matrix techniques (Parry *et al* 1991). Moreover, for this problem it is known (Parry 1992c) that in fluctuation-dominated regimes there is a kind of superuniversality; the scaling function $U(z/D)$ takes the simple form $[\sin(\pi z/D)]^{\vartheta-1}$, where the single parameter ϑ is a critical exponent defined for the equivalent problem in a semi-infinite geometry.

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In this paper we report the results of Monte Carlo simulations for a variety of membrane models in two and three dimensions. We demonstrate that the PDFs exhibit the expected scaling behaviour. Surprisingly, the scaling functions appear to adopt an identical form to those known to be true for the interface models.

The plan of the paper is as follows. In section 2 we present a brief discussion of the surface models which we discuss. Then in section 3 we review the theoretical motivation for our study and discuss the scaling properties of the probability distribution. In section 4 we present the results of Monte Carlo simulations of two- and three-dimensional surfaces. Finally in section 5 we draw some brief conclusions.

2. Models

An interface separates two phase-separated coexisting phases. In general, a microscopic description of the interface is difficult. Modern studies of fluctuation-related interfacial phenomena employ a collective coordinate which models the interfacial position using some suitable receipe (Fisher and Jin 1991). A more comprehensive review of these and related issues can be found in Fisher (1988).

Within this paradigm, the interfacial statistical mechanics may then be determined by summing over all surface configurations Γ, assuming that for each Γ the energy is given by the *interface Hamiltonian* $\mathcal{H}_s(\Gamma) = \gamma A(\Gamma)$. The problem remains of how to specify the interface configuration Γ. In a geometry in which the interface is confined between parallel walls it is not too implausible to suppose that the major contribution will come from configurations in which the interface is almost parallel to the walls. The configuration Γ is then determined by the (single-valued) interface position $z(x, y)$. The interface Hamiltonian is then given by

$$\mathcal{H}_s = \gamma \int \mathrm{d}A = \gamma \int \mathrm{d}^{d-1}S\,[1 + (\nabla_\perp z)^2]^{1/2} \tag{1}$$

where $\nabla_\perp$ denotes differentiation in the plane of the walls, and the integration is carried out over the $d-1$ dimensions in the plane of the confining walls. In the limit of an almost flat interface, and ignoring the contribution of the completely flat interface, this reduces to

$$\mathcal{H}_s = \tfrac{1}{2}\gamma \int (\nabla_\perp z)^2\,\mathrm{d}^{d-1}S. \tag{2}$$

This model has been widely used to discuss interface critical phenomena. There are a number of important assumptions to note. The interface is supposed not to curl back on itself (or, more technically, no *overhangs* are allowed). Then there is no further structure associated with the interface, such as droplets of the wrong phase close to the interface—or rather this is included in the surface tension quantity. In addition, no importance is attached to *curvature* of the interface. And finally a quadratic approximation is made for the excess surface area.

In fact the Hamiltonian of equation (2) is often studied in a discrete approximation, in which the surface is defined on a $(d-1)$-dimensional lattice, and characterized by a discrete set of heights $z = na$, where a is taken to be a molecular length. The statistical mechanical model allows $1 \leqslant n \leqslant D^*$. The quantity D^*a is then a measure of the distance between

the walls. The relation between D^* and D, where Da is the *actual* distance between the walls will be discussed further in section 4. The model now becomes

$$\mathcal{H}_s = \tfrac{1}{2}\sigma \sum_{\substack{\{i,j\} \\ \text{nn}}} (z_i - z_j)^2 \tag{3}$$

where the height is now defined at lattice points i, nn denotes that the sum is taken over nearest-neighbour sites on the lattice, $\sigma = \gamma/h^2$, and h is the length scale of the lattice unit cell. In this form the model is usually known as the *solid on solid*, or SOS, model (Leamy *et al* 1975, Beijeren 1977). We have studied the two-dimensional version of the SOS model, even though it has been intensively studied by other authors (e.g. see Burkhardt 1981, Chui and Weeks 1981, Hemmer and Lund 1988), in order to test the reliability of our method, and we denote this model as model A. We denote the three-dimensional version of this model as model B. In everything that follows we may take $a = h = 1$ without loss of generality.

Over the last few years, stimulated in particular by the work of Helfrich and collaborators (e.g. see Helfrich 1973, 1974, 1990), there has been considerable interest in the behaviour of lipid bilayer membranes in biophysical contexts. The statistical mechanics of membranes is dominated not by surface tension effects (the membranes are in fact rather incompressible, and in any case are made of a different material than the surrounding fluid), but rather by *membrane curvature*. The Hamiltonian is now

$$\mathcal{H}_M = K \int c(r)^2 \, d^{d-1}S \tag{4}$$

where c is the mean curvature at the point r on the membrane. If one adopts the same criteria to define membrane configurations as in the interface case, and makes the same 'nearly flat' approximation, then $c(r) = \nabla_\perp \cdot \hat{n}$, where $\hat{n}$ is a unit vector perpendicular to the membrane, and $\hat{n} = -\nabla_\perp z$, (Helfrich 1990) leading to an SOS membrane Hamiltonian:

$$\mathcal{H}_M = \tfrac{1}{2}K \int (\nabla_\perp^2 z)^2 \, d^{d-1}S. \tag{5}$$

A discrete lattice approximation can be obtained for this Hamiltonian in an analogous fashion to the approximation of equation (3) to equation (2). We shall denote this model as model C, with $d = 2$, and as model D when $d = 3$.

Finally we consider more realistic approaches to the Hamiltonian (4). Leibler *et al* (1987) have introduced a model membrane (in $d = 2$) consisting of beads. These beads are attached to each other in the sense that adjacent beads may not approach more closely than a distance a, but may not be more than a distance $1.8a$ away from each other. It is worth noting, of course, that in the $d = 2$ limit a membrane and a polymer are the same thing. These beads *self-avoid*, but nevertheless overhangs, which are forbidden within the SOS models, are now allowed. We label this model as model E. In this model there is no explicit curvature (at least in the most primitive case which we have considered), but there is a minimum, molecular, length scale below which the membrane cannot bend. This plays a similar role to the correlation length induced by the curvature.

The analogous three-dimensional system is a *bead net* or *polymerized* membrane. This has been studied by Kantor *et al* (1986) and by Gompper and Kroll (1991). Unfortunately, the universal regime of interest to us is not computationally accessible in this case.

Finally in this section we note that the statistical mechanics of all these models is derived by calculating the partition function $\mathcal{Z}$, which is a functional integral of $\exp(\mathcal{H}/k_B T)$ over

configurations Γ, where k_B is Boltzmann's constant and T is the absolute temperature. The relevant parameters in the statistical mechanics of the models described above are thus $\tilde{\gamma} = \gamma/k_BT$, (models A and B) and $\tilde{K} = K/k_BT$ (models C and D). In what follows below we shall, however, drop the tildes.

3. Scaling of the probability distribution

In this section we briefly recall the theoretical arguments which lead to the known scaling properties of the PDF. We consider a membrane or interface model which undergoes an unbinding transition at reduced temperature $t = 0$ and bulk field $h = 0$ (Fisher 1988). At a second-order fluctuation-dominated transition, length scales perpendicular to the wall scale with the single perpendicular correlation length $\xi_\perp$. It is then natural to suppose (Parry 1992a, b) that the PDF has the scaling form

$$p(z) = |t|^{\nu_\perp} \Xi_\pm(z|t|^{\nu_\perp}, h|t|^{-\Delta}) \tag{6}$$

where $\nu_\perp$ and Δ are the perpendicular correlation length and gap critical exponents, respectively. In writing equation (6) we have suppressed the non-universal metric factors and ignored the dependence on any irrelevant fields present. The subscript $\pm$ refers to the cases $t > 0$ (partial wetting) and $t < 0$ (complete wetting), respectively.

The important behaviour of the function Ξ close to the wall is characterized by the short distance expansion (SDE) critical exponent ϑ defined by $\Xi(x, 0) \sim x^{\vartheta-1}$, in the limit $x \to 0$. The values of the SDE exponent are different in the different scaling regimes (Lipowsky and Fisher 1987). In the strong fluctuation regime (Parry 1991a)

$$\vartheta = \tau - 1/\nu_\perp(\tau) \tag{7}$$

whereas in the weak fluctuation regime (Parry 1991b)

$$\vartheta = \tau + 1. \tag{8}$$

The parameter τ in equations (7) and (8) was introduced by Lipowsky and Fisher (1987) in order to characterize the degree of interfacial wandering and will be discussed below. We also note that in $d = 2$ the exponent ϑ is non-universal in the intermediate fluctuation regime (Parry 1991b), reflecting the presence of marginal long-range forces.

Within the approximate nonlinear renormalization group (RG) theory of wetting transitions, Lipowsky (1988) has shown that, for the class of effective Hamiltonians with a fluctuation term $\mathcal{H}[z] = \int \mathrm{d}R\,(\nabla^n z)^2$, the fixed points are controlled by a single parameter $\tau = 2(d-1)/(2n+1-d)$. Consequently the critical exponent $\nu_\perp$ (and hence ϑ) is only dependent on τ itself. The exponent relations (7) and (8) have been fully confirmed for the interface ($n = 1$) and membrane ($n = 2$) models in $d = 2$ (Parry 1991b, Maggs *et al* 1989).

We now turn our attention to the properties of a confined interface. Purely repulsive walls result in a binding potential with no attractive relevant term. Thus confinement between two impenetrable purely repulsive walls—which merely exclude the possible interfacial configurations—can be argued to correspond to finite-size effects at a weak-fluctuation regime wetting transition. A scaling theory of $p(z)$ in this situation, which generalizes equations (6), requires a new argument $\zeta = z/D$ in the scaling function. The associated metric of this finite-size variable is unity. It thus follows (Parry *et al* 1991), so

long as irrelevant model-specific variables are considered, that the PDF when $t = h = 0$ contains a universal scaling contribution $p(z) \cong U(\zeta)/D$. The universal function $U(\zeta)$ will depend on the nature of the surface (interface or membrane), the dimensionality and the fluctuation regime. Moreover, from the RG theory we expect that the SDE of $U(\zeta)$ will depend only on the single variable τ. The SDE for $U(\zeta)$, for confinement between purely repulsive walls, is therefore $U(\zeta) \sim \zeta^{\tau}$ for interfaces and membranes. We should remark, of course, that such scaling behaviour is only expected for sufficiently low dimensionalities. Specifically, for $n = 1$, we only expect scaling for $d < 3$, while for $n = 2$, we expect scaling for $d < 5$ (Fisher 1986).

In addition to the above general RG and scaling predictions for the properties of $U(\zeta)$, there is also a very surprising feature known to be true for the case of interfaces in two dimensions (Parry 1992c). As we have seen above, for interface confinement in a strip exactly at a fluctuation-dominated wetting transition, the PDF $p(z)$ has the simple parametrization

$$p(z) \sim (\sin(\pi\zeta))^{\vartheta(\tau)-1} \tag{9}$$

for all fluctuation regimes, or equivalently, all values of $\vartheta(\tau)$. The result (9) holds in all fluctuation regimes provided the one-body binding potential appearing in the interfacial Hamiltonian is conformally mapped from the semi-infinite geometry.

The existence of a simple parametrization in the case $d = 2$, $n = 1$ is not explicable using the RG theory alone. It can be shown, however, that the universal form (9) is consistent with a conformal invariance hypothesis for the PDF $p(z)$ for systems which maintain their translational invariance in one dimension under the mapping (Parry 1992a). The conformal invariance hypothesis also rederives other known universal parametrizations which appear in the properties of the spectrum of the transfer matrix (Parry 1992c). We postpone further discussion of this rather interesting observation to the final section.

This review of the scaling properties of the PDF provides the background for our study. This paper describes a simulation study of the various effective Hamiltonians described in section 2. In particular, we investigate the resulting PDF, and investigate whether it obeys the scaling form $p(z) = U(\zeta)/D$, and, if so, whether this scaling form is the generalized sinusoidal form given by equation (9). In principle, once $U(\zeta)$ is known for a given Hamiltonian, the associated SDE exponent $\vartheta - 1$ can be determined. In practice, however, this quantity is very difficult to extract with any confidence. Instead we have focused on the global properties of the function $U(\zeta)$.

Our goal in this paper has been very modest: we have sought to examine how the universal parametrization of equation (9), known to be appropriate for $d = 2$, $n = 1$, breaks down for values $d \neq 2$ and $n \neq 1$. However, the appropriate value of ϑ has not been extracted from an SDE analysis of the scaling function $U(\zeta)$. Our fits of the numerical data to a scaling function of the form of equation (9), even when the fit is very good, only therefore allow us to extract an effective exponent. As we shall see, however, the agreement with the data achieved using this single-parameter fit is quite remarkably good and not at all expected. This is discussed in the next section.

4. Monte Carlo simulations

All the simulations have been carried out using the standard Metropolis Monte Carlo algorithm. Except where there are special features of interest we shall not discuss the

method further. A criterion for whether the thermodynamic limit has been reached can be obtained by measuring the variance W of $p(\zeta)$ as a function of system dimension L, where

$$W = \frac{\langle \Delta z^2 \rangle}{D^2}. \tag{10}$$

Clearly we expect the thermodynamic limit to have been reached when W no longer exhibits system size dependence. In simulations of the lattice models A–D, we have used periodic boundary conditions in order to avoid unnecessary edge effects. Typical simulations for the lattice models A–D involved 10^6 Monte Carlo moves per lattice site, with an equilibration period of 2×10^5 moves.

We discuss first the interpretation of the results of the lattice models A–D. We compare with $p(\zeta) \sim [\sin(\pi\zeta)]^{\vartheta-1}$. The effective exponent ϑ can be extracted by plotting $\ln[p(\zeta)]$ against $\ln[\sin(\pi\zeta)]$. This should yield a graph with gradient $\vartheta - 1$. In fact the transformation from z to ζ is not as simple as it seems because for a given set of boundary conditions the effective width D is in fact an empirical quantity; the independent variable setting the width—a kind of 'bare width'—is the quantity D^*. In addition, because the true position of the wall is unknown, the quantity z is itself determined only to within an additive factor a_0 which places the wall with respect to the first possible occupied lattice position of the interface. In model A, the interface in $d = 2$, for which an exact solution of the statistical mechanics exists, $\vartheta - 1 = 2$, $D = D^*$ and $a_0 = 1$†.

The question remains of how to determine the true width D. This can be determined by fitting $p(z)$ close to a wall to the form $C(z + a_0)^{\vartheta-1}$, where C is a constant, ϑ is the (same) exponent, and a_0 is the fitting parameter designed to decide exactly the position of the wall. We recall in this context that, in the limit of small δ, $\sin(\pi\delta/D) \sim \delta$. The position z is here *necessarily* measured with respect to the first possible *occupied* set of lattice sites in the direction between the walls. If ϑ obtained by the two methods agrees, then this is evidence that the assumed form of $p(\zeta)$ is correct. In fact it is difficult to distinguish between different trial functions when there is a free parameter, because $\sin(\pi\delta/D)$ will differ from δ except in the limit of large D for which large simulations are necessary for reliable results. In practice, we use this 'short distance expansion' not so much to determine ϑ, but rather to determine a_0, which allows the correct interpretation of the $p(z)$ results over the whole thickness.

In the case of model A, the short distance expansion, carried out for $D = 20$, using $p(z)$ at the three sets of lattice points closest to the walls, yields $a_0 = 0.91$, and $\vartheta - 1 = 1.806$. The value of $a_0 = 0.91$, is, we believe, consistent with its exact analytic value of 1, in view of the uncertainties in the short distance expansion discussed above. We note that there are no error bars on these values (despite them obviously being in error!) because only three values of $p(z)$ have been used to generate three parameters a_0, ϑ and C. A plot of $\ln[p(\zeta)]/\ln[p(0.5)]$ versus $\ln[\sin(\pi\zeta)]$ for $L = 1000$, $\sigma = 1.0$, using results from $D = 6$, 10, 20 (and taking $a_0 = 1$) yields a value of $\vartheta - 1 = 2$ with a minute error, and a clearly universal curve, as one expects. We have found it useful to exhibit the results of *all* the lattice models together. These data are shown in figures 1 and 2: in figure 1 we show the log–log plots which formally establish the values of the exponent ϑ in the various different

† The *a priori* expectation is that $a_0 \cong 1$, for then the wall is identified with the first non-occupied set of lattice sites. Another inherently plausible value for a_0 is 0.5. This would correspond to the sites representing the centres of the sampling boxes, with particles living on an underlying continuum. In this case the edge of the strip is identified with the edge of the box neighbouring the wall.

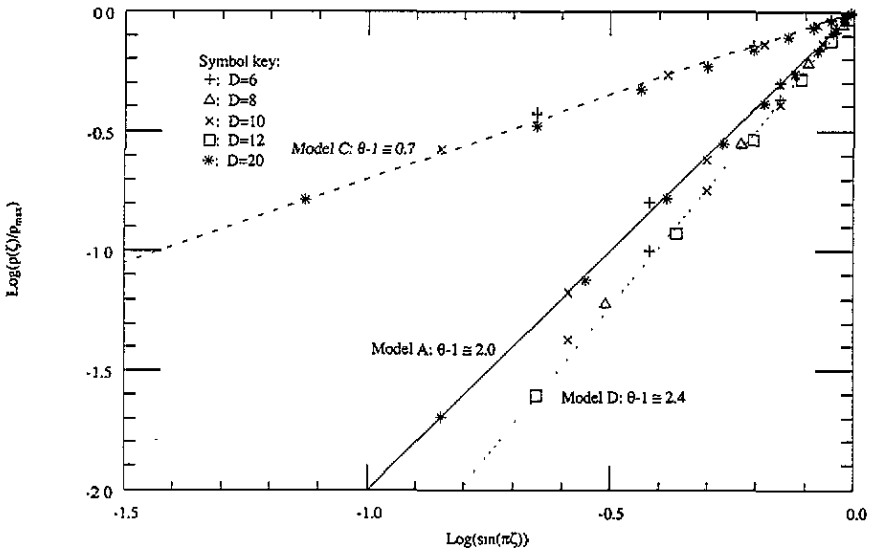

Figure 1. Logarithmic plot of $p(\zeta)$ versus ζ for models A, C and D. In model A we have taken $\sigma = 1.0$; for models C and D, $\kappa = 0.2$.

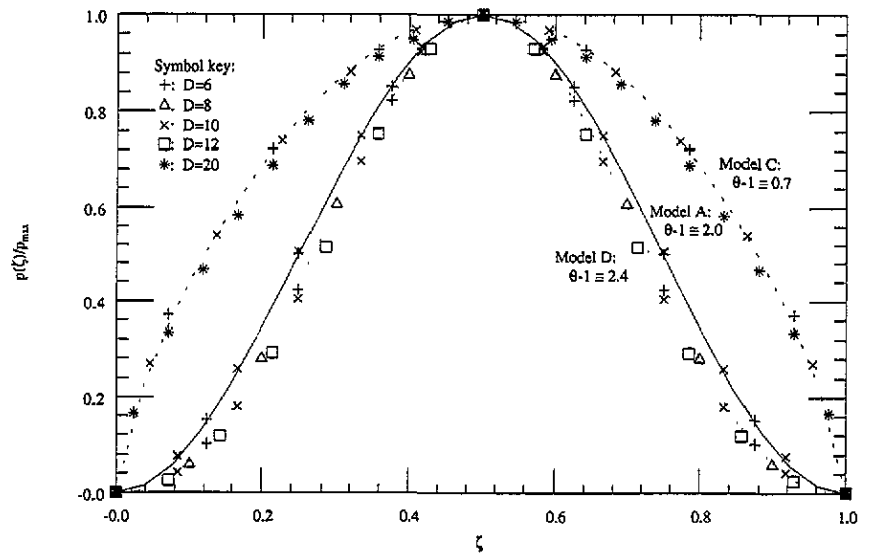

Figure 2. Plot of actual $p(\zeta)$ for the data shown in figure 1.

models, while in figure 2, we show the *actual* $p(\zeta)$; the difference between the results for different exponents ϑ is very striking.

It is also of some interest to note how the thermodynamic limit is approached. In the limit $\sigma \to 0$ the interface positions at neighbouring x are uncorrelated; we therefore expect that in this case the interface can take any position with equal probability; this corresponds formally to $\vartheta = 1$. As σ is increased at fixed system size L, the correlation length $\xi \sim \sigma^{-1}$

decreases until $\xi \ll L$, at which point the crossover to $\vartheta = 3$ is complete. We have monitored this crossover using the parameter W of equation (6); the result is shown in figure 3.

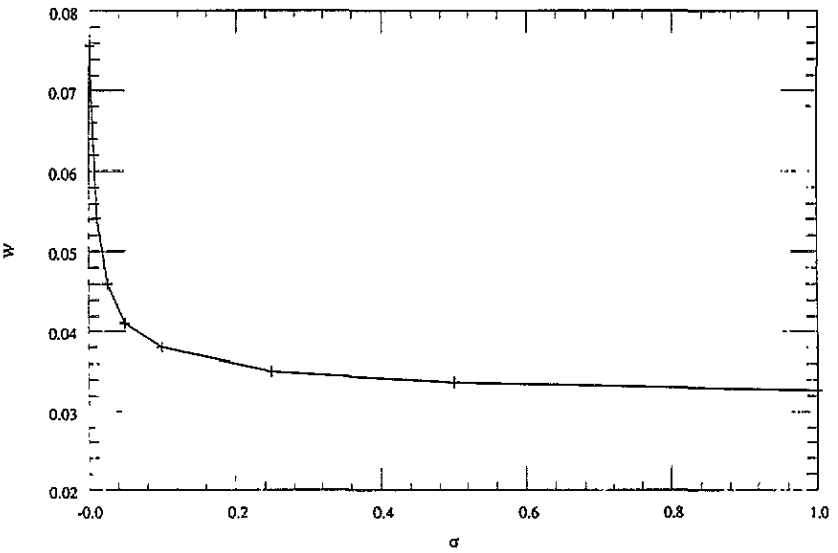

Figure 3. Model A: crossover of variance W of $p(\zeta)$ with increasing surface tension σ, for fixed $L = 1000$, $D = 20$.

We only briefly discuss model B, the interface in $d = 3$. This is the marginal dimension for this model, and so no scaling behaviour is expected, or indeed observed.

An interesting feature of the results from model C (the SOS membrane in two dimensions), where once again we do recover universal behaviour, is that we find $a_0 = 0.67$; in fact we have plotted $p(\zeta)$ in figures 1 and 2 here using $a_0 = 0.5$. We find from these plots that $\vartheta - 1 = 0.72 \pm 0.05$, well below the $\vartheta - 1 = 2$ value for model A. This value is consistent with the value of 0.67 derived from the scaling argument of section 3. For model C we also find that the universality is only *approached* in the limit of large D (although in practice D does not have to be very large); this seems to be in contrast to model A, for which it is *exact*. This can be seen in figure 4, in which we compare the D dependence of the variance W of $p(\zeta)$ in models A and C.

As we have observed in the last section, although model B is not expected to exhibit universal behaviour, it is expected for model D (the SOS model in $d = 3$). In this case we found $a_0 = 0.97$ (consistent with 1), and $\vartheta - 1 = 2.4 \pm 0.1$. We show in figure 5 how a stable value of W is approached as the system dimension L is increased.

We turn now to model E. This is the only truly off-lattice model that we are considering. This model is equivalent to a self-avoiding random walk in a rectangular strip. In the absence of the self-avoidance, the problem can be treated exactly (at least in the limit of $a \ll D$), using Green's function methods commonly in use in polymer physics (Doi and Edwards 1986). One expects a universal $p(\zeta)$, with $\vartheta = 3$. We investigate here the effect of including self-avoidance.

Simulations of this model, because it is off-lattice, and because it is necessary to include an algorithm which tests for self-avoidance, are considerably more time-consuming than other simulations we discuss. In addition, the equilibration time is significantly increased by the self-avoidance, and the approach to universality as a function of polymer length is

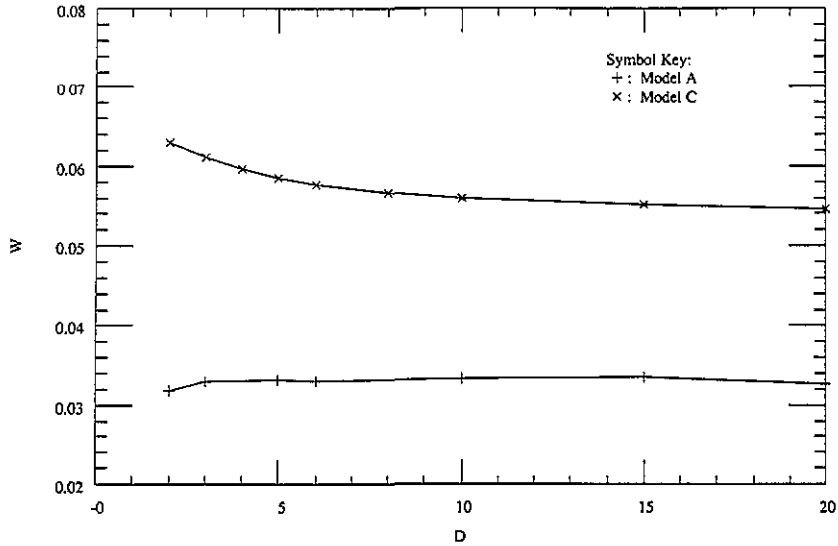

Figure 4. Models A and C: D dependence of W, showing that for model C the universal limit is only approached in the large-D limit.

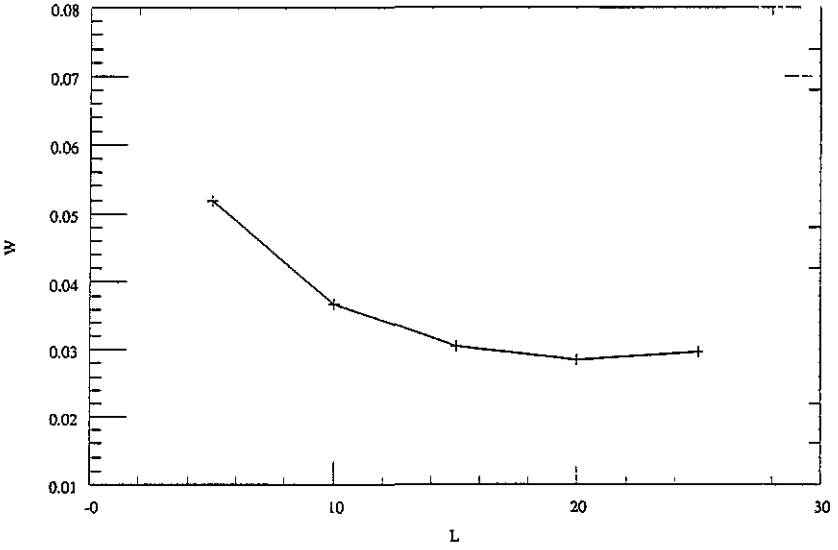

Figure 5. Model D: graph of $W(L)$ showing the approach to the universal limit in the large-L regime; $D = 10.0$, $\kappa = 0.2$.

also slow. In the other simulations we are able to make use of periodic boundary conditions. In model E, by contrast, we have to allow the ends to be free, thus presumably increasing finite-size effects, which can only be eliminated by considering very long polymers. Finally, it is difficult to derive $p(\zeta)$ accurately because a finite grid must be adopted in order to sample the probability. Too fine a grid leads to inevitable fluctuations, while too large a grid fails to sample the distribution function sufficiently well. On the other hand, because the model is off-lattice, we no longer have the problem of defining the width D in terms of a bare width D^*. However, one should bear in mind that the *physical* width of the strip is always $D + a$, if the centres of the beads are confined to a strip of width D.

The criteria which we use for gauging the extent to which the universal regime has

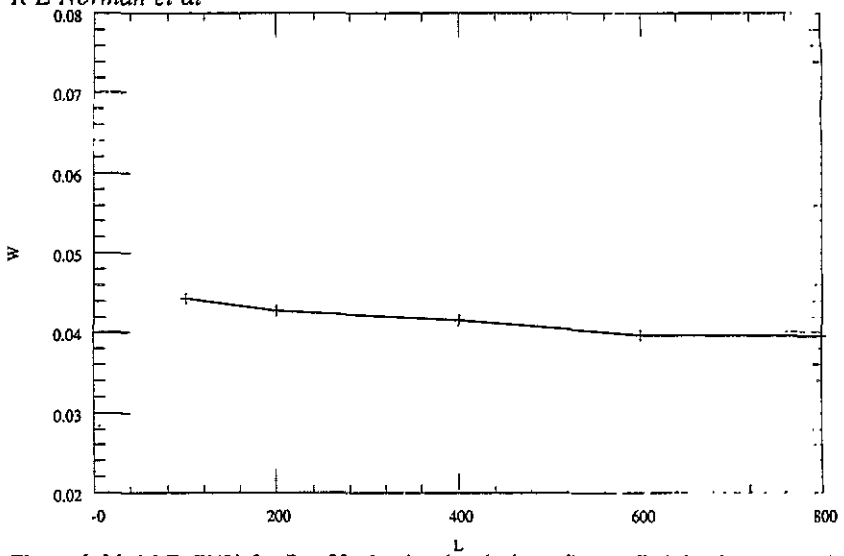

Figure 6. Model E: $W(L)$ for $D = 20$, showing that the large-distance limit has been reached.

been reached, as system size is increased, are as follows: (a) the variance W seems to have settled down, (b) the end-to-end length of the polymer has settled down, and (c) the power law ϑ_W, derived from the behaviour of $p(z)$ close to the wall, has settled down.

In figure 6 we show the dependence of the variance W as the polymer length L is increased. At $L = 800$, the largest value for which we took measurements, $W = 0.0395 \pm 0.0005$. In figure 7 we examine the effective exponent ϑ_W as a function of L. Above $L \cong 400$ this seems to settle down at a value of 2.5, although in fact the $p(z)$ corresponding to this have not in fact settled down. Interestingly, if we suppose that $p(\zeta) \sim [\sin(\pi\zeta)]^{\vartheta-1}$, the value of W which we derive corresponds to $\vartheta - 1 \cong 1.44 \pm 0.05$, which is reassuringly consistent with ϑ_W. The results are at least consistent with the canonical form for the probability distribution function.

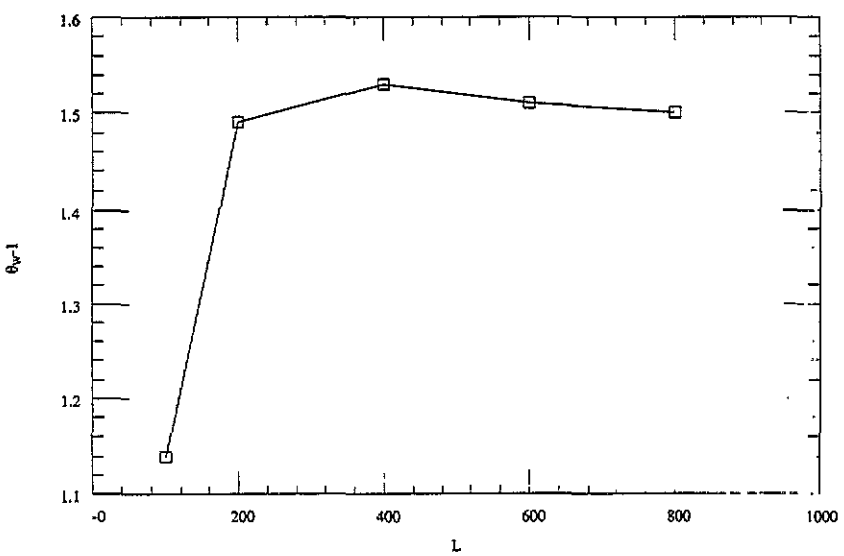

Figure 7. Model E: the effective SDE exponent $\vartheta_W(L)$, showing the large-distance limit has been reached.

Finally we mention that we attempted to follow the same programme with a three-dimensional bead net model, but found that the universal regime (if it exists) was beyond the limits of the computational resources available to us.

5. Discussion and conclusions

In this paper we have given results of a Monte Carlo study of the scaling behaviour of confined interfaces and membranes. We have found that membranes, whether off- or on-lattice, and whether in $d = 2$ or $d = 3$, seem to obey a scaling law for their PDF of a form originally predicted to obtain only for interfaces in $d = 2$. While one of us (Parry 1992a) has shown that the predicted superuniversal form for interface PDFs in $d = 2$ is consistent with conformal invariance argument, the deeper reason for this result remains unclear. Preliminary investigation by Henkel and Parry (1994) suggest that the conformal hypothesis is strictly limited to interface models in $d = 2$, and should therefore not apply to the membrane models in any dimensionality. Nevertheless, from a computational point of view the success of the parametrization suggested by the conformal invariance hypothesis is remarkable.

In $d = 2$, the value of the effective (on-lattice) membrane exponent $\vartheta - 1$ is numerically 0.72, but this is within the experimental error of the result predicted from a scaling argument of 2/3 (model C). Whether we can indeed ascribe the difference between these two figures to numerical uncertainty is unclear, however, for in $d = 3$ (model D), the predicted scaling value is 2.0, whereas we find numerically $\vartheta - 1 = 2.4$. The off-lattice result (model E) of $\vartheta - 1 = 1.5$ remains intriguing but at this stage completely without explanation.

Further analytical studies of the applicability of the conformal invariance hypothesis seem possible. For the interface problem the success of the hypothesis seems to be related to the conformal invariance of an appropriate classical Euler–Lagrange equation with marginal forces. For the two-dimensional membrane it may also be possible to make progress in solving the Fokker–Planck equation for the partition function in the confined geometry. The three-dimensional membrane clearly requires further study, in particular to find why the fitted exponent differs so dramatically from the scaling prediction. At present the most likely explanation is that some subtle numerical analysis is required to extract the short-distance expansion exponent, and that the superuniversal scaling form, although good, is fortuitous away from the wall.

Finally, further numerical studies of the off-lattice models, both in two and three dimensions, would produce interesting and more reliable results.

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